tolerance. For example, the earth subtends 16.4° to the desired edges of coverage from synchronous altitude. The designer wishes to have as much gain as possible, so he initially sets $B/C = 1$, where $B = 16.4°$ and the maximum full coverage signal gain is 20.1 db. If we refer to Fig. 2, we see that this results in a 3-db loss at the edge for perfect pointing and a 7.6-db loss for 25%, or 4.1° pointing error. With the given pointing error (25%), Fig. 4 shows that the designer would be better off to open the beamwidth to 1.40 times the coverage (or to 23°) and accept a 6.4-db maximum loss (presenting a minimum of 13.7-db signal gain at the edges).

Another application of interest is the case of lunar vehicle communication to the earth, where $C \cong 2°$. For moving vehicles using tracking antennas, the pointing error probably will equal or exceed half the coverage angle, perhaps a 50% or 1° error. Figure 4 shows that the minimum losses occur for $B/C = 1.75$. Thus, a 3.5° beamwidth antenna would be selected for lunar communication if a 50% pointing error existed and all other factors affecting beamwidth were assumed attainable and practical. [At DSIF frequencies, a 3.5° ($B/C = 1.75$) beamwidth antenna would require an 8-ft dish.] The expected minimum gain is 34 db minus 8.8 db (as obtained from the ordinate of Fig. 4 opposite the minimum of the $Ep = 50\%$ curve), or 25.2 db. This is the number that would be used in link calculations and communications system design.

The curves are completely general (within the constraints discussed) and can be used in reverse, that is, they can be used to determine coverage angles where specified signal gains are to be exceeded for given antennas and pointing errors. Thus, given any three of the variables, the optimum value of the fourth can be determined readily.

Meteorological Instrumentation for the Boosted-Arcas Rocket Probe

Glenn Cato*
Electro-Optical Systems, Inc., Pasadena, Calif.

Nomenclature

g = acceleration of gravity
k = Boltzmann constant
m = mean molecular weight
T = absolute temperature
μ = absorption coefficient
ρ = density

Introduction

The U.S. Naval Missile Center, Point Mugu, Calif., has recently developed a boosted version of the Arcas sounding rocket. The Sidewinder 1A rocket motor is used as the booster. This note describes this rocket probe and suggests measurement techniques and instruments that might be used with it. A complete description of work accomplished on the project is given in Ref. 1. Because the apogee is well above parachute altitudes, instrumentation was sought that would operate during the ascent leg of the trajectory.

The Sidewinder-boosted-Arcas sounding rocket is shown in Fig. 1. It carries the standard Arcas nose cone, which is 20.1 in. long, has an 11.5-in. extension, and provides a payload compartment of about 300 in.³ Typical trajectories are shown in Fig. 2.

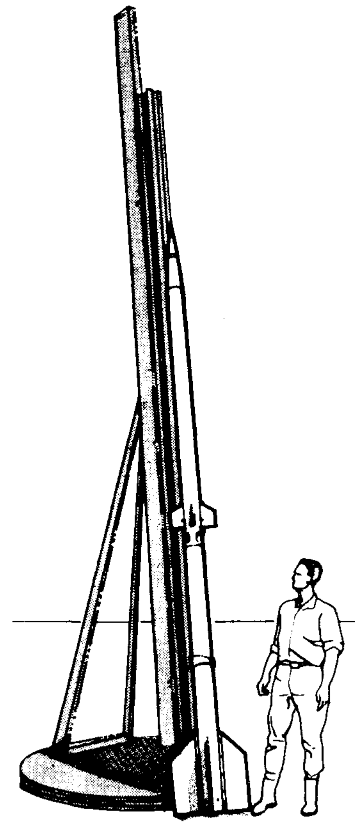

Fig. 1 Sidewinder-boosted-Arcas rocket probe (from photograph supplied by U. S. Naval Missile Center, Point Mugu, Calif.).

Density

Attention was given to the spectrometric technique for the measurement of density. Photometric instrumentation has the advantage of being compact, and the technique has been used successfully with large rockets.

Basically, the instrument measures the intensity and the reduction in intensity of the sunlight due to the absorption of specific solar wavelengths as a function of altitude. Since the amount of absorption depends on the total mass of air above the sensor, it is possible to derive the following relation for density in terms of received energy:

$$\ln\frac{J_2}{J_1} = \mu\int_{h_1}^{h_2} \rho\, dh \tag{1}$$

where J_1 and J_2 are the measured intensities of sunlight at altitudes h_1 and h_2, respectively.

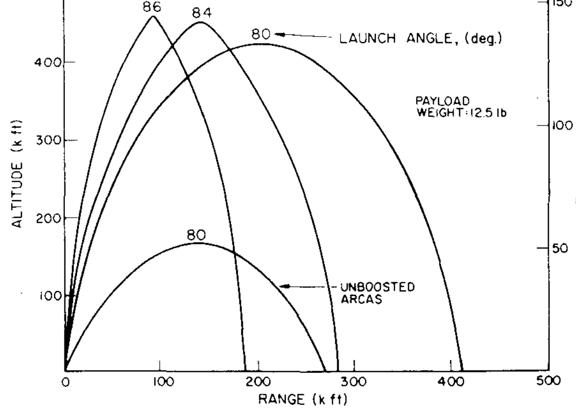

Fig. 2 Sidewinder 1A: Arcas trajectories (data supplied by U. S. Naval Missile Center, Point Mugu, Calif.).

Presented as Preprint 64-320 at the 1st AIAA Annual Meeting, Washington, D. C., June 29–July 2, 1964; revision received October 13, 1964. Work sponsored by Department of the Navy, Navy Purchasing Office, Los Angeles, Calif., Contract No. N123(61756)32689A(PMR) for the U. S. Naval Missile Center, Point Mugu, Calif.

* Aeronomy Projects Supervisor. Member AIAA.

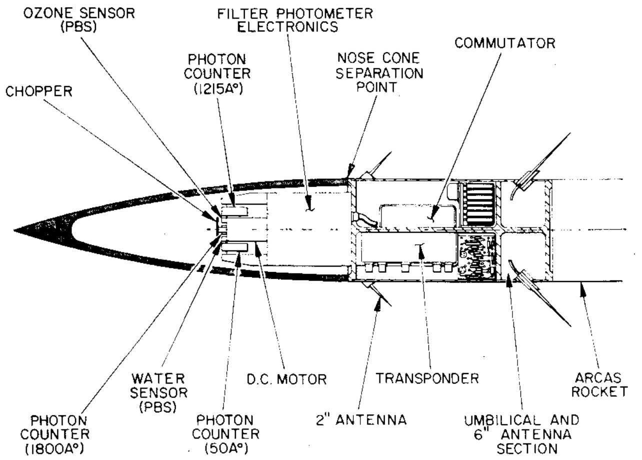

Fig. 3 Filterphotometer composite payload configuration.

Since only the ratio of the intensities appears in the equation, absolute calibration of the instrument is unnecessary. Stability of calibration, however, is of great importance. Outgassing should have a negligible effect on the measurement because the total mass of atmosphere above the sensor far exceeds the mass of the outgassed layer immediately adjacent to the instrument detector.

The choice of wavelength depends upon the depth to which the radiation can penetrate the atmosphere and on the atmospheric constituent responsible for the absorption. Below 85 km, it is necessary to select a wavelength that has a greater penetration depth. At 0.18 μ, the absorption is sensitive to the density of molecular oxygen. Absorption in this band can be used to determine the total density up to about 90 km, since up to this altitude turbulent mixing keeps the molecular oxygen concentration constant and the amount of dissociation is small. To cover the range of altitudes from 70 to 150 km, the instrument should contain at least two, and probably three separate sensors. An 0.18-μ detector would operate between 70 and 90 km; a 0.1215-μ detector between 90 and 125 km. A third detector, sensitive around the 0.005-μ region, would be required to extend the altitude range to 150 km. A single instrument package, as is depicted in Fig. 3, could contain all three detectors. The nose cone would be jettisoned at an altitude of $\sim$ 60 km.

An alternative to the photometer is the ionization gage for the determination of particle density at high altitudes. Before accurate knowledge of density at the high end of the altitude range of interest can be obtained with ionization gages, a method is required for obtaining molecular mass and number of the constituent gases. A small Omegatron is the type of device needed, but it still requires development. The chief factor that makes the use of the ionization gage very difficult, or of any gage carried entirely aboard a rocket probe, is outgassing from the rocket and the instruments.

Ozone

At rocket probe altitudes, it is suggested that an adaptation of the Paetzold filterphotometric technique to measure ozone concentration be used; the sun is used as the standard light source, and the attenuation of the solar ultraviolet by the ozone is measured. The change in attenuation of the sunlight in an ozone absorption band between two altitude levels is a measure of the ozone content of the interval.

In the past, experimenters have used an optical system with a narrow field of view. Sun-tracking devices have been used to keep the instrument locked onto the sun, but this is not practical for a simple rocket such as the boosted Arcas. It is proposed, instead, that an optical system with a wide field of view be used and that the rocket be allowed a moderate pitch and yaw (0°–15°). Because the degradation follows the cosine law, for off-axis angles of 15° or so, the reduction or error should be less than 5%.

Water Vapor

To measure water vapor from aboard a rocket probe, only a differential absorption technique that utilizes solar spectrometry appears to have a sufficiently rapid response. For our purposes, either the 1.37-μ or 1.25-μ absorption band is suitable, with the 1.37-μ band being slightly superior due to its greater depth. The longer wavelengths could be utilized if indium arsenide sensors were employed. However, these sensors still are in the developmental stage.

The chief advantage of the use of the differential absorption technique is that the instruments can be very small, and the density, water vapor, and ozone filterphotometers could be combined into a single payload, as shown in Fig. 3.

To provide the unattenuated reference, an additional wavelength must be chosen which is a window where the absorption by ozone, water vapor, or the atmosphere in general is nil. The output of the water or ozone sensors is, in each case, compared to that of the reference sensor, which is sensitive in a transmission band. Thus, relative intensities, rather than absolute intensities, can be used. Also, in this way, the error due to the sun not being directly on the instrument axis is cancelled out, since misalignment affects all sensors equally.

Each sensor requires a separate dielectric interference filter. Our analysis indicates that the reference filter should center at 0.5 μ and have a bandpass of 0.1 μ. The ozone filter should center on 2.54 μ and have a bandpass of 0.05 μ. Water vapor should be monitored with a filter centered on 1.37 μ and have a bandpass 0.05 μ wide.

Temperature

The method deemed most applicable in the altitude range of 70 to 150 km is to measure the density and to calculate the gas kinetic temperature. This can be done by means of the familiar relationship

$$\rho_2 = \rho_1 \exp[-mgh_2/kT] \qquad (2)$$

If the ionization gage technique is used to measure density, the molecular weight will be available. If not, it can be assumed to remain unaltered up to about 80 km. Above 80-km altitude, the mean molecular weight would have to be estimated, which should cause an error of only 10–20%. At levels above 100 km, the concept of electron temperature becomes important. Electron temperature can be measured directly and, by means of certain simplifying assumptions, it may be used to calculate kinetic temperature. For this reason, consideration has been given for the development of a Langmuir probe for the boosted Arcas. A Langmuir probe could be flown alone, or it could be part of an integrated payload with an ionization gage to measure density. This configuration would yield density, kinetic, and electron temperatures.

Pressure

For the boosted Arcas, the best onboard, pressure-measuring technique is deemed to be a pitot-static tube. The most expeditious course would be to use a conventional pitot tube with the nose tip patterned after the flow from a point source in a uniform stream. A pitot-static tube for the boosted Arcas is currently undergoing flight test at the Pacific Missile Range, Calif.

There is some contention that a pitot tube with a source-shaped tip is not satisfactory for the transitional and free-molecule flow regions above $\sim$ 80 km. Here, a short, open-channel tube with the leading edge internally chambered at about 15° has been proposed.

Probably the best sensor to use with a pitot tube is the diaphragm gage. This gage has a faster response than the

Pirani gage and, unlike ionization gages, it is immune to chemical decomposition of the air, which begins about 90 km. Recently, Electro-Optical Systems, Inc. has investigated the adaptation of a solid-state strain gage as a pressure transducer. Deflection of the diaphragm causes a strain in the silicon crystal elements fastened to its surface. The piezoresistive effect induced in the crystal by the strain is a measure of the pressure force, causing the diaphragm to deflect.

Reference

1 Cato, G., Marlow, D. G., and Snyder, L. M., "Study of meteorological instrumentation and telemetry systems suitable for 70 to 150 km altitude," Electro-Optical Systems, Inc. Rept. 3800-Final, Navy Contract N123(61756)32689A for Pacific Missile Range, Armed Services Technical Information Agency Doc. AD-601483 (March 1963).

Manned Vehicles as Solids with Translating Particles: I

C. F. Harding*
Douglas Aircraft Co., Inc., Santa Monica, Calif.

MEN moving within a space vehicle are extensible objects; furthermore, the motion of the vehicle feeds back on their ability to execute a desired path. It is assumed that they are small relative to the vehicle, whereas the latter is somewhat quiescent, but in the purely mathematical situation, there is no restriction placed on spin. We thus study the formal problem of a spinning rigid body within which a given number of mass points slide along grooves in a prescribed manner. It is desired to determine the effect of these particle motions on the rotation of the body when no external torques exist.

The equations of motion for a system of interacting rigid bodies are first derived from basic principles. An argument similar to that of Abzug[1] is given, but the final result is cast in a form different from his. By referencing all translations to the composite center of mass, we get an expression with no reference to inertial space displacement. In this way we construct inertia dyadics that depend only on variables defined onboard the body. The form of these dyadics suggests the kinds of particle constraints that can lead to equations similar to free-body motion. In particular, the case of a body of revolution, in which all particles remain on the symmetry axis, has a straightforward solution: the motion of an axially symmetric body with constant mass and time-varying longitudinal moment of inertia. We end the text with a detailed example for the rotation of a symmetric body having one particle whose relative motion is sinusoidal.

General Equation

Let an indefinite number of rigid bodies P^i be moving about in space, and choose one as the main body with a reference frame B^0 attached to it. Let $\boldsymbol{\beta}^i$ be the displacement of the mass center (c.m.) of P^i from B^0, $\boldsymbol{\omega}_0{}^i$ be the angular velocity of P^i relative to B^0, and $\mathbf{I}^i$ be the inertia dyadic of P^i about its own c.m. The equation of translation for P^i relative to space-fixed axes S is then

$$m^i \frac{d^2}{dt^2}(\boldsymbol{\sigma} + \boldsymbol{\beta}^i) = \mathbf{f}^i + \sum_{j\neq i} \mathbf{f}^{ij}$$

where $\boldsymbol{\sigma}$ is the displacement of the origin of B^0 from the origin of S, whereas $\mathbf{f}^i$ and $\mathbf{f}^{ij}$ are, respectively, the total external force and the force due to P^j acting on P^i. The equation of rotation for P^i, relative to space-fixed axes S, is as follows:

$$\frac{d}{dt}\left[\mathbf{I}^i\cdot(\boldsymbol{\omega} + \boldsymbol{\omega}_0{}^i) + (\boldsymbol{\sigma} + \boldsymbol{\beta}^i) \times m^i \frac{d}{dt}(\boldsymbol{\sigma} + \boldsymbol{\beta}^i)\right] = \mathbf{l}^i + (\boldsymbol{\sigma} + \boldsymbol{\beta}^i) \times \mathbf{f}^i + \sum_{j\neq i}[\mathbf{l}^{ij} + (\boldsymbol{\sigma} + \boldsymbol{\beta}^i) \times \mathbf{f}^{ij}]$$

where $\boldsymbol{\omega}$ is the angular velocity of B^0 relative to S, whereas $\mathbf{l}^i$ and $\mathbf{l}^{ij}$ are, respectively, the total external torque and the torque due to P^j acting on P^i about its center of mass. It must be true from both equations that

$$\sum_{i=0}^{n}\sum_{j\neq i} \mathbf{f}^{ij} = 0$$

$$\sum_{i=0}^{n}\sum_{j\neq i} [\mathbf{l}^{ij} + (\boldsymbol{\sigma} + \boldsymbol{\beta}^i) \times \mathbf{f}^{ij}] = 0$$

for if not, and all $\mathbf{f}^i$, $\mathbf{l}^i$ were zero, then the linear and angular momenta of the system relative to S would not be constant. If now, $\mathbf{c}$ is the center of mass of the system in terms of S, then

$$\boldsymbol{\sigma} + \boldsymbol{\beta}^i = \mathbf{c} + \boldsymbol{\beta}^i - \frac{1}{m}\sum_{j=0}^{n} m^j\boldsymbol{\beta}^j = \mathbf{c} + \mathbf{r}^i$$

where m is the total mass and $\mathbf{r}^i$ is the displacement of the P^i center of mass from $\mathbf{c}$. We next rewrite two expressions from the rotation equations by summing over the bodies

$$\sum_{i=0}^{n}\frac{d}{dt}\left[(\boldsymbol{\sigma} + \boldsymbol{\beta}^i) \times m^i \frac{d}{dt}(\boldsymbol{\sigma} + \boldsymbol{\beta}^i)\right] = \mathbf{c} \times m\frac{d^2}{dt^2}\mathbf{c} + \sum_{i=0}^{n} \mathbf{r}^i \times m^i \frac{d^2}{dt^2}\mathbf{r}^i$$

$$\sum_{i=0}^{n}(\boldsymbol{\sigma} + \boldsymbol{\beta}^i) \times \mathbf{f}^i = \mathbf{c} \times \mathbf{f} + \sum_{i=0}^{n} \mathbf{r}^i \times \mathbf{f}^i$$

where $\mathbf{f}$ is the sum of $\mathbf{f}^i$. Finally, on summing all the equations of translation, cross multiplying on the left by $\mathbf{c}$, and subtracting from the summed rotation equations, one gets

$$\sum_{i=0}^{n}\frac{d}{dt}[\mathbf{I}^i\cdot(\boldsymbol{\omega} + \boldsymbol{\omega}_0{}^i)] + \sum_{i=0}^{n}\mathbf{r}^i \times m^i\frac{d^2}{dt^2}\mathbf{r}^i = \sum_{i=0}^{n}\mathbf{l}^i + \sum_{i=0}^{n}\mathbf{r}^i \times \mathbf{f}^i$$

so that retaining just the main body, reducing the rest to points, and setting external influences to zero

$$\frac{d}{dt}(\mathbf{I}\cdot\boldsymbol{\omega}) + \sum_{i=0}^{n}\mathbf{r}^i \times m^i\frac{d^2}{dt^2}\mathbf{r}^i = 0 \qquad (1)$$

with $\mathbf{I} = \mathbf{I}^0$.

With respect to the body,

$$d(\mathbf{I}\cdot\boldsymbol{\omega})/dt = \mathbf{I}\cdot\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{I}\cdot\boldsymbol{\omega} \qquad (2a)$$

$$d^2\mathbf{r}^i/dt^2 = \ddot{\mathbf{r}}^i + 2\boldsymbol{\omega} \times \dot{\mathbf{r}}^i + \dot{\boldsymbol{\omega}} \times \mathbf{r}^i + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}^i) \qquad (2b)$$

Now we have as follows:

$$\mathbf{r}^i \times (2\boldsymbol{\omega} \times \dot{\mathbf{r}}^i) = -2\mathbf{r}^i \times (\dot{\mathbf{r}}^i \times \boldsymbol{\omega}) = -2(\mathbf{R}^i\cdot\dot{\mathbf{R}}^i)\cdot\boldsymbol{\omega}$$

where

$$\mathbf{R}\cdot\dot{\mathbf{R}} = -(\mathbf{r}\cdot\dot{\mathbf{r}})\mathbf{E} + t\mathbf{r}$$

and $\mathbf{E}$ is the idemfactor. Similarly,

$$\mathbf{r}^i \times (\dot{\boldsymbol{\omega}} \times \mathbf{r}^i) = -(\mathbf{R}^i\cdot\mathbf{R}^i)\cdot\dot{\boldsymbol{\omega}}$$

while ($\boldsymbol{\Omega}$ is formed similar to $\mathbf{R}$)

$$\mathbf{r}^i \times [\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}^i)] = \mathbf{r}^i \times (\boldsymbol{\Omega}\cdot\boldsymbol{\Omega})\cdot\mathbf{r}^i = \mathbf{r}^i \times \boldsymbol{\omega}\boldsymbol{\omega}\cdot\mathbf{r}^i = -\boldsymbol{\omega} \times \mathbf{r}^i\mathbf{r}^i\cdot\boldsymbol{\omega} = -\boldsymbol{\omega} \times (\mathbf{R}^i\cdot\mathbf{R}^i)\cdot\boldsymbol{\omega}$$

Received August 4, 1964; revision received December 2, 1964.

* Research Engineer, Stability and Control Theory Branch, Advance Flight Mechanics Department, Missile and Space Systems Division.